Math 126 Summer 2026 HW 5

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Problem 1: [Maximum Failure]

Create an example to show that the wave equation

$$\begin{aligned} \partial_t^2 u - \partial_x^2 u &= 0 \\ u_x(a) = u_x(b) &= 0 \end{aligned}$$

fails to satisfy a weak maximum principle on $[0,\infty) \times (a,b)$ (Hint: superposition of waves).

Problem 2: [RH Condition Solutions]

Recall that we proved a piecewise-defined weak solution u would satisfy a certain condition along its piecewise-boundary. Suppose $u \in L^1_{loc}$ is a function so

$$u = \begin{cases} u_- & x < \sigma(t) \\ u_+ & x > \sigma(t) \end{cases}$$

for some classical solutions u_- and u_+ to $\partial_t v + \partial_x q(v) = 0$ (q is C^1) and some curve$(t, \sigma(t))$ satisfying

$$\sigma'(t)(u_+ - u_-) = q(u_+) - q(u_-)$$

where the classical solutions also match boundary conditions

$$g(x) = \begin{cases} g_- & x < x_0 \\ g_+ & x > x_0 \end{cases}$$

for $\sigma(0) = x_0$. Prove that u is a weak solution to

$$\begin{aligned} \partial_t u + \partial_x q(u) &= 0 \\ u(0,x) &= g(x) \end{aligned}$$

Hint: use the RH-condition proof.

Problem 3: [RH Condition Application]

Find a weak solution to Burgers' equation $\partial_t u + u\partial_x u = 0$ with initial condition

$$u(0,x) = \begin{cases} 1 & x < -1 \\ 0 & x \in (-1,0) \\ 2 & x > 0 \end{cases}$$

Draw a picture of the shock curves and regions of definition.

Problem 4: [Weak Solutions]

Create a suitable definition for the weak solution of the PDE

$$\begin{cases} \partial_t^2 u + 2a\partial_t u + a^2 u - \partial_x^2 u = 0 & [0,\infty) \times (a,b) \\ (u, u_t)|_{t=0} = (g, 0) \\ u(a) = u(b) = 0 \end{cases}$$

Find a weak solution provided $g \in L^1_{loc}((a,b))$.
Problem 5:
Use Integration-by-parts to compute the distributional limit

$$\lim_{j\to\infty} j^2|x|\cos(jx)$$

Problem 6:
Find the Greens' function on $\{(x,y) \in \mathbb{R}^2 \mid x,y > 0\}$.